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One-sided versus two-sided readings of *many* and *few*

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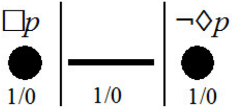
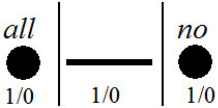
Aims of this talk:

- discuss Béziau's (unpublished LNAT1) proposal to transpose his results on the logical geometry of the modal logic S5 to that of the subjective quantifiers *many* and *few*
- propose an alternative analysis of *many* and *few*, which seems to fare equally well from a strictly logical perspective, but which we argue to be more in line with certain linguistic desiderata
- compare the two analyses in terms of two scales:
  - scale of semantic complexity
  - scale of lexical complexity
- compare the two analyses in terms of the types of Aristotelian diagrams they generate

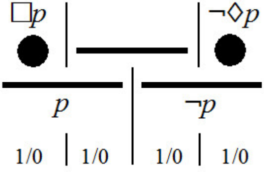


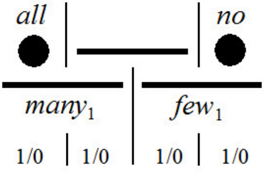


S5-formula	bitstring	FOL-quantifier
$\Box p$	100	*all*
$\neg\Box p$	011	*not all*
$\neg\Diamond p$	001	*no*
$\Diamond p$	110	*at least one*
$\Box p \vee \neg\Diamond p$	101	*no or all*
$\neg\Box p \wedge \Diamond p$	010	*some*

some	$\equiv$	*at least one but not all*	$010 = 110 \wedge 011$	**two-sided**
at least one	$\equiv$	*some or all*	$110 = 010 \vee 100$	**one-sided**

Béziau's one-sided readings of "many" and "few"





S5-formula	bitstring	FOL-quantifier
$\Box p$	1000	*all*
$\neg\Box p$	0111	*not all*
$\neg\Diamond p$	0001	*no*
$\Diamond p$	1110	*at least one*
$\Box p \vee \neg\Diamond p$	1001	*no or all*
$\neg\Box p \wedge \Diamond p$	0110	*some*
p	1100	$many_1$
$\neg p$	0011	few_1

level	S5-formula	bitstring	subjective quantifier
L2	p	1100	*many*$_1$
	$\neg p$	0011	*few*$_1$
L1	$p \wedge \neg\Box p$	0100	*many*$_1$ *but not all*
	$\neg p \wedge \Diamond p$	0010	*at least one but few*$_1$
L3	$\neg p \vee \Box p$	1011	*all or few*$_1$
	$p \vee \neg\Diamond p$	1101	*no or many*$_1$
L2	$\Box p \vee (\neg p \wedge \Diamond p)$	1010	*all or (at least one but few*$_1$*)*
	$\neg\Box p \wedge (p \vee \neg\Diamond p)$	0101	*no or (many*$_1$ *but not all)*

The conjunctions *many*$_1$ *but not all* and *at least one but few*$_1$ create the L1 elements 0100 and 0010 by excluding the extreme values of the tripartition, i.e. *all* (1000) and *no* (0001), respectively.

- entailments in S5
  - from L1 'necessity' (1000) to L2 'actual truth' (1100)
  - from L1 'impossibility' (0001) to L2 'actual falsehood' (0011)
- analogous entailments for subjective quantifiers
  - from L1 *all* (1000) to L2 $many_1$ (1100)
  - from L1 *no* (0001) to L2 few_1 (0011)
- suppose that John has read all three books in the universe of discourse
  - *John has read all books* is obviously true
  - *John has read many books* is very likely to be considered false ('three books' does not really count as 'many books')
- suppose that John has read none of the books in the univ. of discourse
  - *John has read no books* is obviously true
  - *John has read few books* is much less obvious (conflict with the existential presupposition of *few*)

- **solution**: two-sided readings for *few* and *many*

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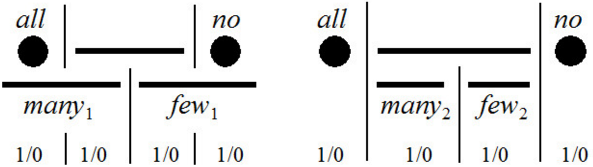
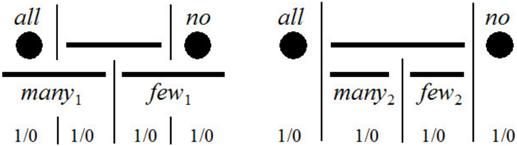


many$_2$	$= 0100 =$	two-sided L1 incompatible with *all*	$= 1000$
few$_2$	$= 0010 =$	two-sided L1 incompatible with *no*	$= 0001$

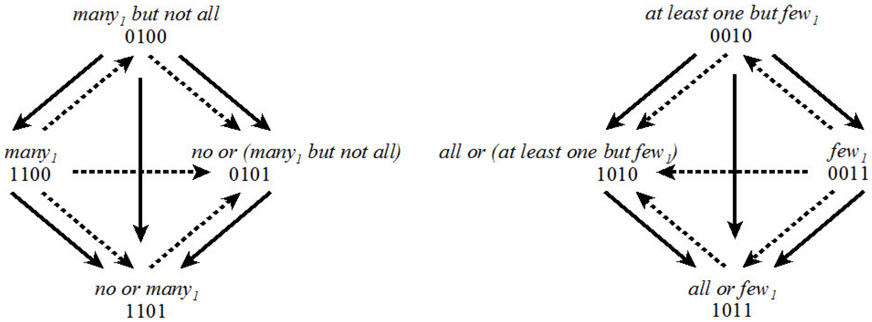
level 2 **disjunctions** = lexically complex expressions,
cfr. English *little or no*; Dutch *weinig of geen* and French *peu ou pas*

many$_2$ *or all*/*many*$_2$ *if not all*	$0100 \vee 1000 = 1100$	$\equiv$	*many*$_1$
few$_2$ *or no*/*few*$_2$ *if any*	$0010 \vee 0001 = 0011$	$\equiv$	*few*$_1$
many$_2$ *or few*$_2$	$0100 \vee 0010 = 0110$	$\equiv$	*some*



level	Béziau's analysis	bitstring	alternative analysis
L2	*many*$_1$	1100	*many*$_2$ *if not all*
	few$_1$	0011	*few*$_2$ *if any*
L1	*many*$_1$ *but not all*	0100	*many*$_2$
	at least one but few$_1$	0010	*few*$_2$
L3	*all or few*$_1$	1011	*all or (few*$_2$ *if any)*
	no or many$_1$	1101	*no or (many*$_2$ *if not all)*
L2	*all or (at least one but few*$_1$*)*	1010	*all or few*$_2$
	no or (many$_1$ *but not all)*	0101	*no or many*$_2$

Semantic vs lexical complexity in the Béziau analysis



- **discrepancies** between:
  - semantic complexity (full line arrows) = entailment L1 > L2 > L3
  - lexical complexity (dashed line arrows) = amount of lexical material
- difference in **orientation** between:
  - the lattices for semantic complexity = from top to bottom
  - the lattices for lexical complexity = from the outside inwards

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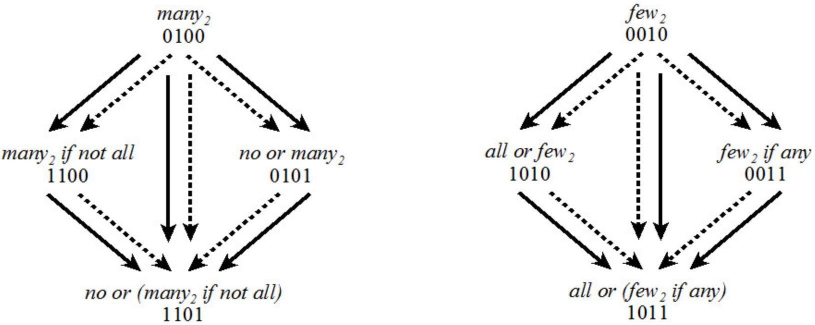
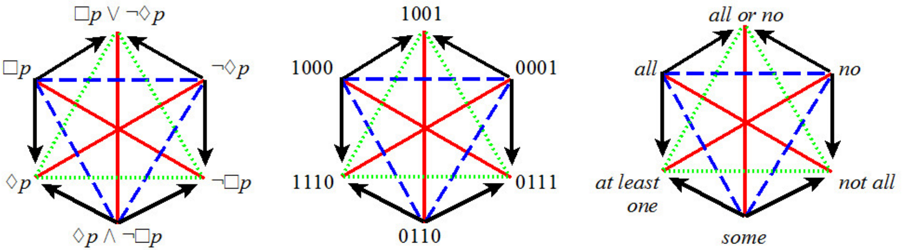
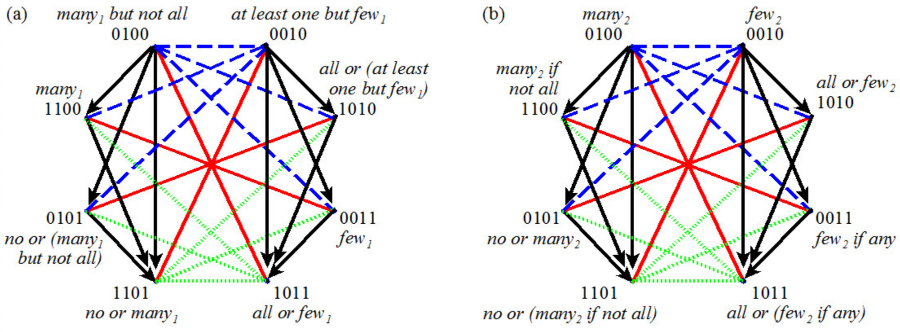


- no more discrepancies between:
  - semantic complexity (full line arrows) = entailment L1 > L2 > L3
  - lexical complexity (dashed line arrows) = amount of lexical material
- parallel **orientation** of:
  - the lattices for semantic complexity = from top to bottom
  - the lattices for lexical complexity = from top to bottom



- **contradiction**: 3 diagonals: 2 x L1-L3 and 1 x L2-L2
- **contrariety**: triangle L1-L2-L1
- **subcontrariety**: triangle L3-L2-L3
- **subalternation**: 6 arrows: 2 x L1-L2, 2 x L2-L3 and 2 x L1-L3

Buridan octagons for “many” and “few”



- **contradiction**: 2 x L1-L3 and 2 x L2-L2 ⇝ ***many$_1$/few$_1$***
- **contrariety**: 1 x L1-L1 and 4 x L1-L2 ⇝ ***many$_2$/few$_2$***
- **subcontrariety**: 1 x L3-L3 and 4 x L2-L3
- **subalternation**: 4 transitivity triangles L1-L2-L3
- **unconnectedness square**: 4 pairs of L2-L2

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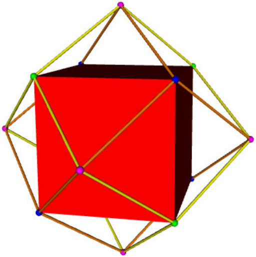
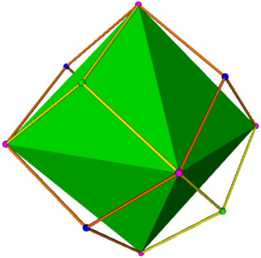
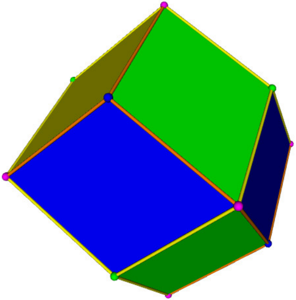


cube
8 vertices
4 x L1
4 x L3



octahedron
6 vertices
6 x L2



rhombic dodecahedron
14 vertices
*L0 *0000
*L4 *1111
$\mathbf{14 = 2^4 - 2 = 16 - 2}$

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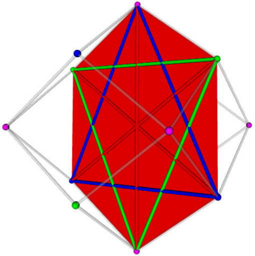
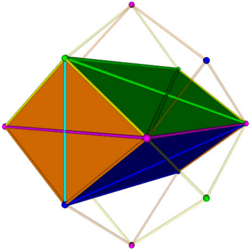
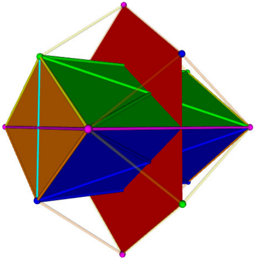
strong JSB hexagon



Buridan octagon



rhombic dodecahedron



rhombicube

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- discussed Béziau's one-sided analysis of *many*$_1$/*few*$_1$ based on the analogy between the modal logic S5 and the subjective quantifiers.
- proposed an alternative, two-sided analysis of *many*$_2$/*few*$_2$, which more adequately reflects the relations of entailment (*all* $\nrightarrow$ *many*, and *no* $\nrightarrow$ *few*) and disjunction (*few if any*, *many if not all*).
- compared the two analyses in terms of discrepancies between the scale of semantic complexity and the scale of lexical complexity.
- compared the two analyses in terms of the types of Aristotelian diagrams they generate: identical strong Jacoby-Sesmat-Blanché hexagons but different Buridan octagons/rhombicubes:
  - **contradiction** for ***many***$_1$***/few***$_1$
  - **contrariety** for ***many***$_2$***/few***$_2$

Thank you!

More info: www.logicalgeometry.org

H. Smessaert & L. Demey, ‘Béziau’s Contributions to the Logical Geometry of Modalities and Quantifiers’. In: A. Koslow & A. Buchsbaum (eds.), *The Road to Universal Logic*, 2015, Springer, pp. 475-493.