One-sided versus two-sided readings of *many* and *few*

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In this presentation, we first discuss Béziau's (unpublished) proposal to transpose his results on the logical geometry of the modal logic S5 to that of the subjective quantifiers *many* and *few*. We propose an alternative analysis of *many* and *few*, which seems to fare equally well from a strictly logical perspective, but which we argue to be more in line with certain linguistic desiderata. The core of Béziau's analysis consists in treating the subjective quantifiers $many_1$ and few_1 on a par with the so-called 'null-modalities' in S5, namely the formulas p and $\neg p$, which do not contain a modal operator. In S5, we start with a tripartition of logical space into 'necessity' (1000), 'contingency' (0110) and 'impossibility' (0001), and superimpose upon it a bipartition into 'actually true' (1100) and 'actually false' (0011). Analogously, the space of quantification can be tripartitioned by means of the expressions *all* (1000), *some but not all* (0110) and *no* (0001). Béziau's analysis now superimposes a bipartition by means of the subjective quantifier expressions $many_1$ (1100) and few_1 (0011). The entailments in S5 from level 1 (L1) 'necessity' (1000) to level 2 (L2) 'actual truth' (1100) and from L1 'impossibility' (0001) to L2 'actual falsehood' (0011) get straightforward counterparts in the realm of subjective quantification. Thus, $many_1$ and few_1 are L2 elements: $many_1$ (1100) is entailed by *all* (1000), whereas few_1 (0011) is entailed by *no* (0001). This accounts for the leftmost and rightmost nodes in Figure 1. The remaining six nodes are then built by means of the Boolean operators of conjunction and disjunction. The conjunctions $many_1$ *but not all* and *at least one but* few_1, e.g., create the L1 elements 0100 and 0010 by excluding the extreme values of the tripartition, i.e. *all* (1000) and *no* (0001), respectively.

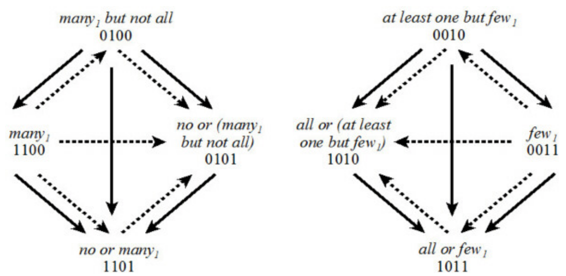

Figure 1

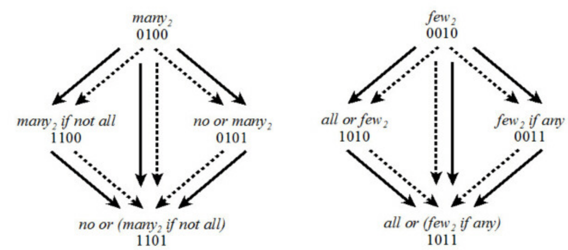

Figure 2

Although Béziau takes *all* (1000) to entail his one-sided L2 element $many_1$ (1100), this entailment needn't hold. For example, in case John has read all three books in the universe of discourse, the proposition *John has read all books* is obviously true, but the proposition *John has read many books* is very likely to be considered false, for the simple reason that 'three books' does not really count as 'many books'. Our alternative analysis reflects this possible absence of entailment by assigning the two-sided L1 reading 0100 to natural language $many_2$. This analysis is further supported by the fact that a lexically complex expression such as *many if not all* exactly allows us to turn the two-sided $many_2$ into a one-sided reading, by incorporating the *all* in the disjunction, thus retrieving the L2 semantics 1100 of Béziau's $many_1$. Analogously, Béziau's entailment from *no* (0001) to few_1 (0011) is somewhat problematic. Concluding from the truth of *John has read no books* to that of *John has read few books* runs into conflict with the existential presupposition (of 'at least one' book having been read) that seems to accompany the latter proposition. Hence, our few_2 receives a two-sided L1 analysis 0010 which is incompatible with *no* 0001. Again, lexically complex expressions such as *few if any* (i.e. *few if not no*) or *little or no*, change the two-sided few_2 into a one-sided reading, by incorporating the *no* in the disjunction. Translational equivalents for this disjunctive semantics are found in Dutch *weinig of geen* and French *peu ou pas*.

The full line arrows in Figures 1 and 2 represent the semantic complexity increasing from L1 at the top to L3 at the bottom, thus reflecting the entailment or subalternation relation. The dashed line arrows, by contrast, reflect the increase in lexical complexity, i.e. the amount of lexical material an expression consists of. Béziau's analysis in Figure 1 has a mismatch between the increases in semantic and lexical complexity in 8 out of the 12 cases: for example, $many_1$ is lexically more primitive than $many_1$ *but not all*, but the former's L2 bitstring (1100) is semantically more complex than the latter's L1 bitstring (0100). Analogously, few_1 is lexically more primitive than *at least one but* few_1, but the former's L2 bitstring (0011) is again semantically more complex than the latter's L1 bitstring (0010). Similar discrepancies can be observed the next level up. In more visual terms, the 'orientation' of the two lattices for semantic complexity in Figure 1 is *from the top downwards*, whereas that of the two lattices for lexical complexity is *from the outside inwards* (i.e. with $many_1$ and few_1 as their respective starting points). The alternative analysis in Figure 2, by contrast, avoids these mismatches between semantic and lexical complexity: the respective lattices share a single ‘orientation’, viz. *from the top downwards*. In a final step, the two analyses are compared in terms of the Aristotelian diagrams they give rise to. The two analyses share a JSB hexagon for the ordinary quantifiers, but complement it with different Buridan octagons/rhombicubes for the subjective quantifiers.