Aristotelian Diagrams for Multi-Operator Formulas in Avicenna and Buridan

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It is well-known that the categorical statements from syllogistics have modal versions, such as "all men necessarily run". The fourteenth-century philosopher John Buridan showed that the Aristotelian relations holding between such formulas do not yield a classical square of oppositions, but rather an octagon (see the work by Stephen Read and others). The Aristotelian relations holding between formulas that involve a quantifier and a modality were already studied by Avicenna in the eleventh century (although he did not actually draw an octagon). Furthermore, it has recently been shown by Saloua Chatti that Avicenna extended this analysis in two directions by considering more fine-grained quantifiers and modalities (such as those in "some but not all men necessarily run" and "all men possibly but not necessarily run") and thereby obtained two 12-formula analyses. In this paper, we will examine how these analyses are connected to each other, and present one further extension, in which all other analyses are integrated. We start by "decomposing" Buridan's octagon into two independent squares: one for the quantifiers (all, some, no, not-all) and one for the modalities (necessary, possible, impossible, not-necessary). The "product" of these squares yields 4x4=16 pairwise equivalent formulas, and is isomorphic to the octagon. Next, we move to the Boolean closure of these squares, by adding two quantifiers (some-and-not-all, all-or-no) and two modalities (possible-and-not-necessary, necessary-or-impossible), thereby obtaining a quantifier hexagon and a modality hexagon, respectively. We now consider the "product" of the quantifier hexagon with the modality square, and that of the quantifier square with the modality hexagon: these consist of 6x4=4x6=24 pairwise equivalent formulas, and correspond exactly to Avicenna's two 12-formula analyses. Finally, one can also consider the "product" of the two hexagons, which consists of 6x6=36 pairwise equivalent formulas, and which subsumes all previous analyses in an octadecagonal diagram.