Metalogical Decorations of Logical Diagrams

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In the last few decades, the classical square of oppositions has been extended to larger diagrams such as hexagons and cubes, and applied to various contemporary logical systems such as modal logic. Recently, logicians have also started using Aristotelian diagrams to visualize the relations between metalogical notions such as tautology and consistency. In this presentation, we will extend this line of work and provide a unifying perspective on the existing results. Next to the set of Aristotelian relations, we define three other sets of logical relations, viz. opposition, implication and duality relations.

It can be shown that every pair of formulas stands in exactly one of the four opposition relations, viz. contradiction (CD), contrariety (C), subcontrariety (SC) and non-contradiction (NCD). Hence, they constitute a quadripartition of logical space, whose powerset consists of 2^4 = 16 elements, and can be visualized by means of a three-dimensional rhombic dodecahedron (RDH). This diagram is significantly larger than the metalogical hexagons studied so far, and moreover, turns out to contain many of them as subdiagrams. For example, several authors distinguish between 'strong' and 'weak' senses of contrariety, which correspond to C and C v CD, respectively. We can define a second 'weak' sense of contrariety, viz. as C v NCD, and prove it to be dual to the first (in the sense that two formulas are in C v CD iff their negations are not in C v NCD). The Aristotelian and duality relations between these various senses of contrariety can be visualized by means of octagons, which are subdiagrams of RDH (see Figures 1 and 2).

Since implication is a partial order, we can construct a second metalogical RDH for the implication relations. This stands in sharp contrast to the well-known hexagon for the arithmetical ordering between numbers, which is a total order.

Figure 1

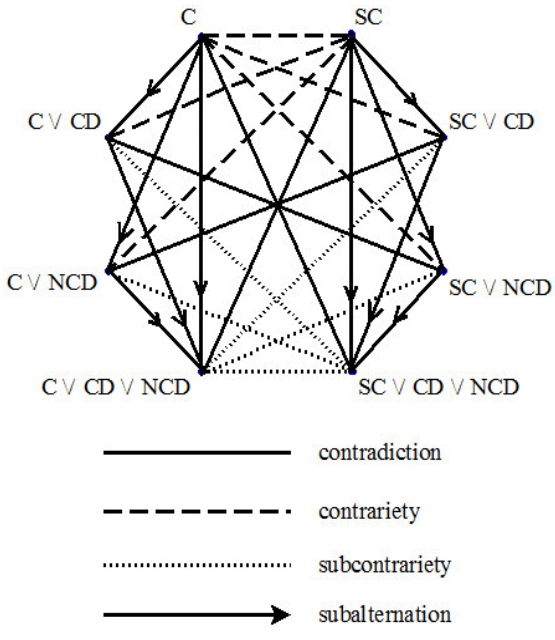

Figure 2