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Duality and Lexicalization in Medieval Squares of Opposition

Lorenz Demey

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- two issues related to the square of opposition
  - Aristotelian vs. duality relations
  - (non-)lexicalization
- each of them **separately** is (relatively) well-understood
- this talk: explore the **interaction** between these two issues
  - argue that they mutually reinforce each other
  - use this interaction to shed new light on some issues in medieval logic
- based on joint work with Hans Smessaert and Dany Jaspers

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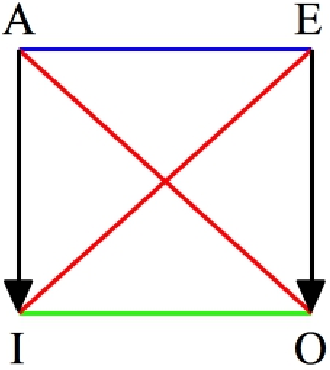
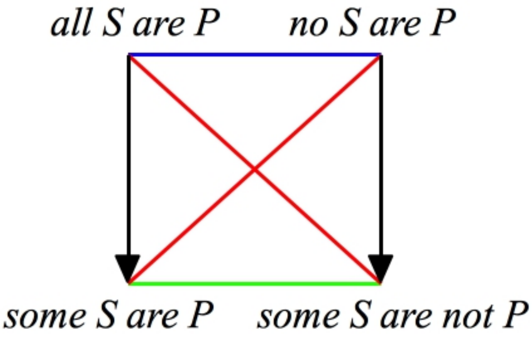
- an Aristotelian diagram visualizes some formulas/expressions and the Aristotelian relations holding between them
- two propositions are said to be

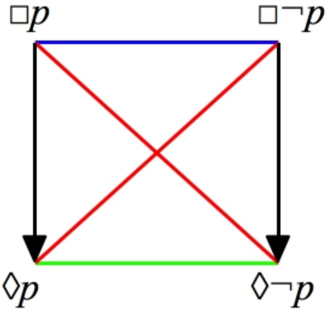
contradictory	iff	they cannot be true together and they cannot be false together,
contrary	iff	they cannot be true together but they can be false together,
subcontrary	iff	they can be true together but they cannot be false together,
in subalternation	iff	the first proposition entails the second but the second doesn't entail the first

Vowel convention for the square of opposition





(assumption of existential import: there exists at least one S)


□p
□¬p
◊p
◊¬p

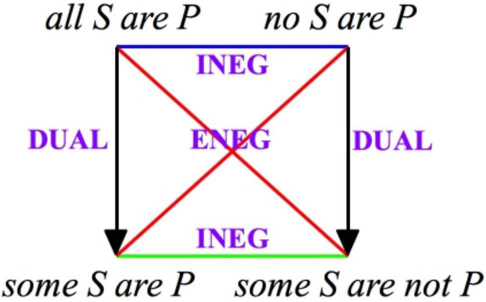
The duality relations

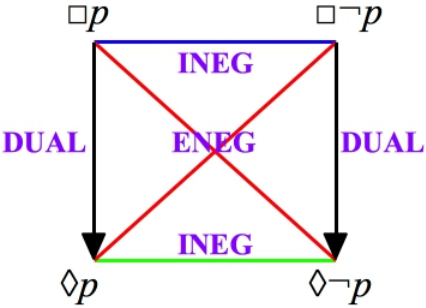
- many Aristotelian diagrams not only exhibit Aristotelian relations, but also duality relations among their elements
- view a proposition φ as the output of some n-ary operator O on some inputs $x_1, \dots, x_n$: $\quad \varphi = O(x_1, \dots, x_n)$
- given two operators O_1, O_2, we say that

O_2 is the **internal negation** of O_1 (INEG)
iff $O_2(x_1, \dots, x_n) \equiv O_1(\neg x_1, \dots, \neg x_n)$

O_2 is the **external negation** of O_1 (ENEG)
iff $O_2(x_1, \dots, x_n) \equiv \neg O_1(x_1, \dots, x_n)$

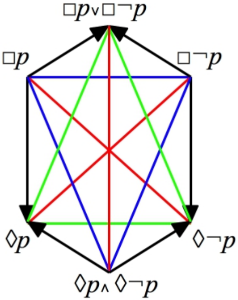
O_2 is the **dual** of O_1 (DUAL)
iff $O_2(x_1, \dots, x_n) \equiv \neg O_1(\neg x_1, \dots, \neg x_n)$

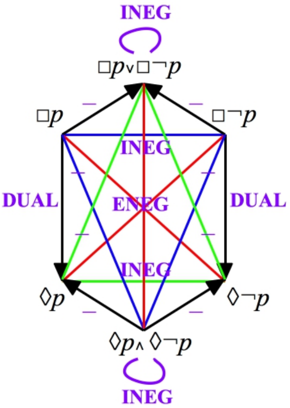

all S are P
no S are P
INEG
DUAL
ENEG
DUAL
INEG
some S are P
some S are not P

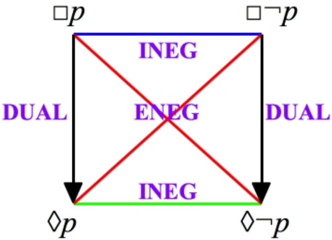

$\Box p$
$\Box\neg p$
INEG
DUAL
ENEG
DUAL
INEG
$\Diamond p$
$\Diamond\neg p$

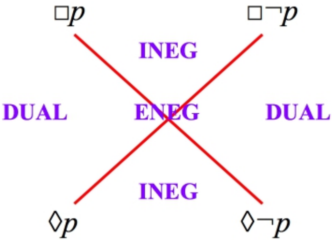
- D: defined for formulas of the form $\varphi = O(x_1, \ldots, x_n)$
  A: defined for **all** formulas
- D: **symmetric**: if $R(\varphi, \psi)$ then $R(\psi, \varphi)$
  A: subalternation is antisymmetric: if $SA(\varphi, \psi)$ then not $SA(\psi, \varphi)$
- D: **deterministic**: if $R(\varphi, \psi_1)$ and $R(\varphi, \psi_2)$ then $\psi_1 \equiv \psi_2$
  A: a formula can have **multiple** contraries
- D: **serial**: for all $\varphi = O(x_1, \ldots, x_n)$, there exists ψ such that $R(\varphi, \psi)$
  A: a formula can have **no** contraries **at all**
- D: four by four: $\{O, \text{INEG}(O), \text{ENEG}(O), \text{DUAL}(O)\}$ (Klein 4-group)
  A: squares, but also hexagons, octagons, etc.
- D: not sensitive to the details of the underlying logical system S
  A: highly **logic-sensitive**: contradictories in S_1, contraries in S_2

- Jacoby-Sesmat-Blanché (JSB) hexagon
- Boolean closure of the square









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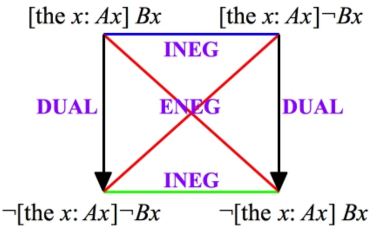
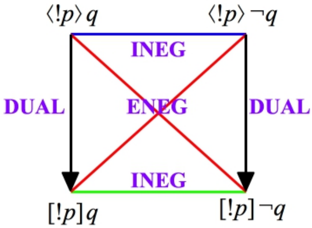
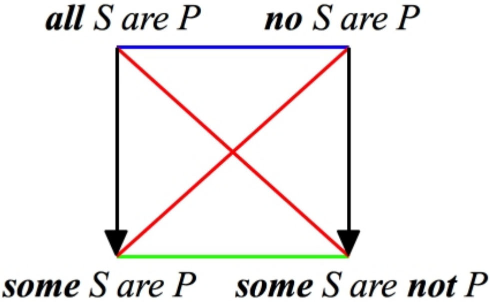
- conceptual independence of Aristotelian and duality relations
- nevertheless: many (all?) squares in the philosophical/logical **literature** are simultaneously Aristotelian squares and duality squares
  - classical examples (cf. middle ages): quantifiers, modalities
  - contemporary examples: definite descriptions, public announcement logic

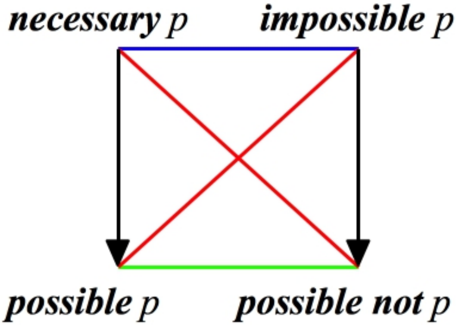




- the A-corner is primitively lexicalized as *all*
- the I-corner is primitively lexicalized as *some*
- the E-corner is primitively lexicalized as *no*
- the O-corner is not primitively lexicalized



- the A-corner is primitively lexicalized as *necessary*
- the I-corner is primitively lexicalized as *possible*
- the E-corner is primitively lexicalized as *impossible*
- the O-corner is not primitively lexicalized



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- not just with quantifiers and modalities,
  but also in **other lexical domains**
  - *all*, *some*, *no* vs. *some not*
  - *necessary*, *possible*, *impossible* vs. *possible not*
  - *everywhere*, *somewhere*, *nowhere* vs. *somewhere not*
  - *everybody*, *somebody*, *nobody* vs. *somebody not*
  - *always*, *sometimes*, *never* vs. *sometimes not*
  - *both*, *either*, *neither* vs. *either not*
- not just in English, but also in **other natural languages**
- first author to point this out: Thomas Aquinas,
  In Arist. De Int. (*Expositio libri Peryermeneias*),
  Book I, Lesson 10

Sicut autem supra dictum est, quandoque aliquid attribuitur universali ratione ipsius naturae universalis; et ideo hoc dicitur praedicari de eo universaliter, quia scilicet ei convenit secundum totam multitudinem in qua invenitur; et ad hoc designandum in affirmativis praedicationibus adinventa est haec dictio, **omnis** [...] In negativis autem praedicationibus adinventa est haec dictio, **nullus** [...]

Quandoque autem attribuitur universali aliquid vel removetur ab eo ratione particularis; et ad hoc designandum, in affirmativis quidem adinventa est haec dictio, **aliquis** vel **quidam**, per quam designatur quod praedicatum attribuitur subiecto universali ratione ipsius particularis; sed quia non determinate significat formam alicuius singularis, sub quadam indeterminatione singulare designat; unde et dicitur individuum vagum. In negativis autem **non est aliqua dictio posita**, sed possumus accipere, **non omnis**

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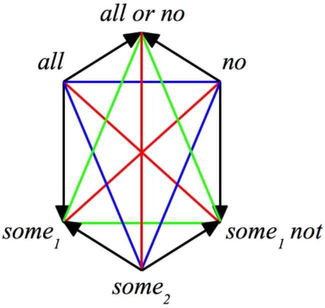
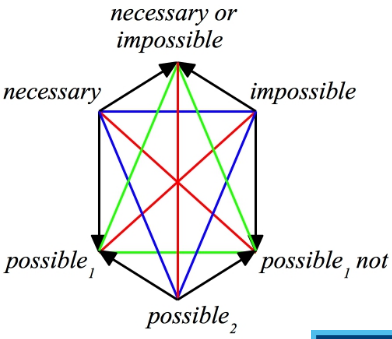
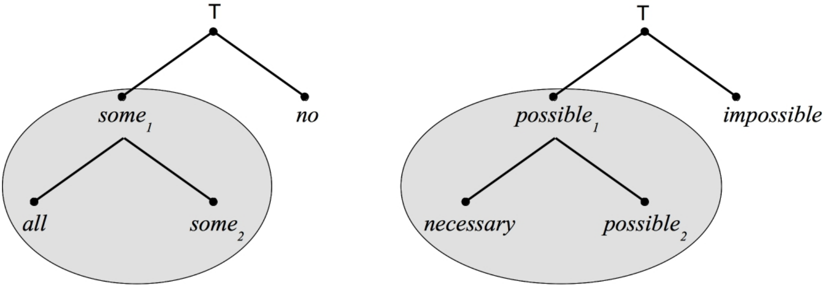
- systematic explanation of the non-lexicalization of the **O-corner**
- Horn: pragmatic (Gricean) account
- Jaspers: JSB hexagon = square + Y-corner (below), U-corner (above)
  - the Y-corner is (often) co-lexicalized with the I-corner
  - the **U-corner** is not lexicalized





- recursive partitioning of the universe
- not lexicalized: disjunction **across** subuniverse
  - quantifier U-corner: *all or no*
  - modal U-corner: *necessary or impossible*
  - quantifier O-corner: $\textit{some}_1$ *not* $\equiv$ $\textit{some}_2$ *or no*
  - modal O-corner: $\textit{possible}_1$ *not* $\equiv$ $\textit{possible}_2$ *or impossible*



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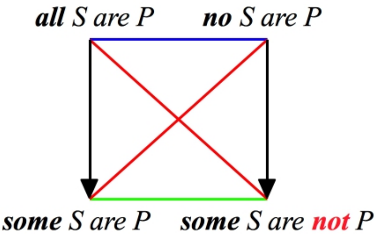
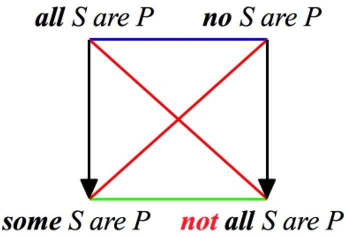
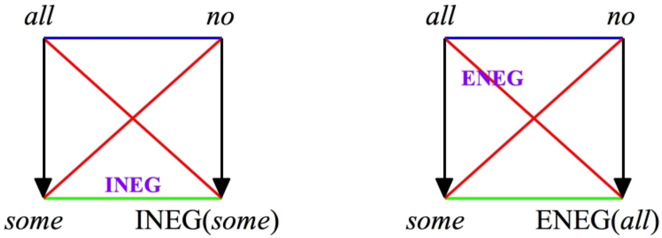
- since the O-corner is not primitively lexicalized, it needs to be expressed in terms of one of the other corners
- in the literature we find at least two versions of the square

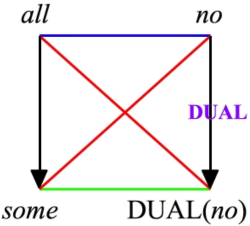


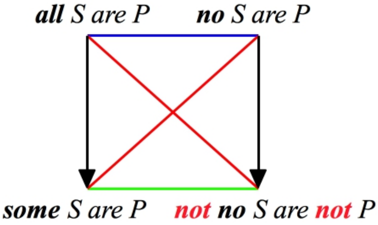


- *some S are not P* = INEG(*some S are P*)
- *not all S are P* = ENEG(*all S are P*)



- O = INEG(I) and O = ENEG(A), but also O = DUAL(E)
- ***not no*** *S are* ***not*** *P*
- cognitive processing difficulties





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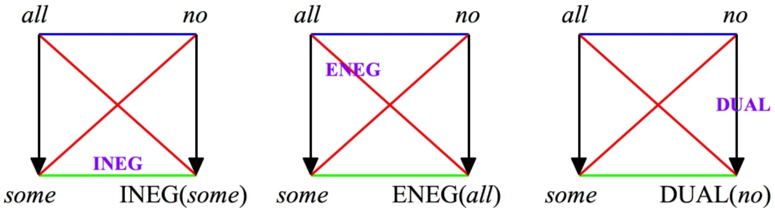
- the O-corner is itself not primitively lexicalized
- but it can be non-primitively expressed in three ways,
  viz. as a duality-theoretic variant of each of the three other corners



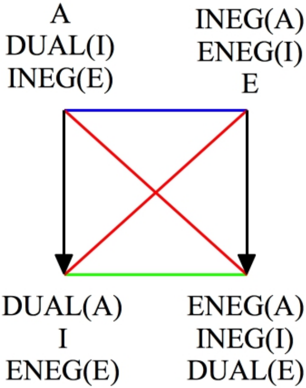
- A is primitively lexicalized as ***all***

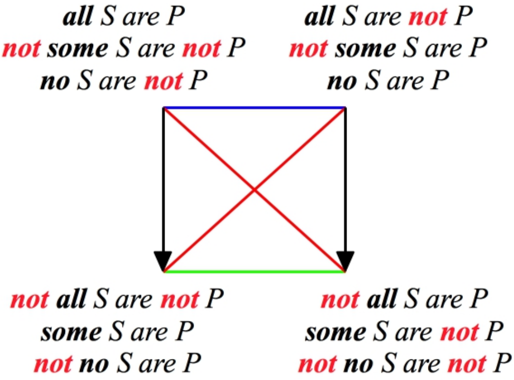
- A = INEG(E)
- E is primitively lexicalized as ***no***
- so A is non-primitively lexicalized as ***no not***

- A = DUAL(I)
- I is primitively lexicalized as ***some***
- so A is non-primitively lexicalized as ***not some not***

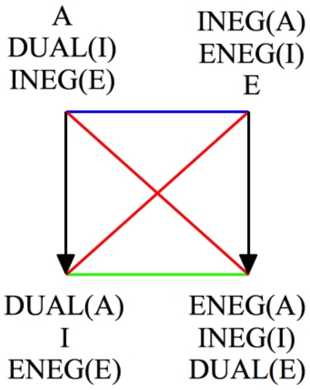
- A = ENEG(O)
- O is itself not primitively lexicalized
- so A gets no additional non-primitive lexicalization

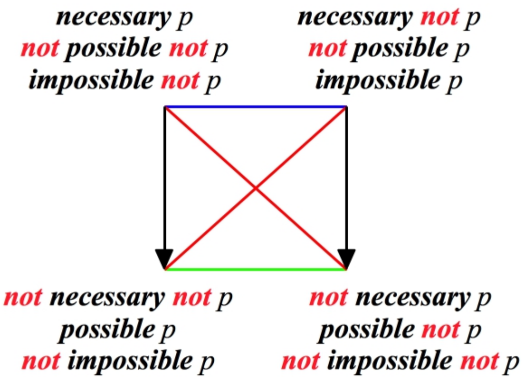
O-corner	INEG(*some*)	*some not*
	ENEG(*all*)	*not all*
	DUAL(*no*)	*not no not*
A-corner	primitive	***all***
	INEG(*no*)	*no not*
	DUAL(*some*)	*not some not*
I-corner	primitive	***some***
	ENEG(*no*)	*not no*
	DUAL(*all*)	*not all not*
E-corner	primitive	***no***
	INEG(*all*)	*all not*
	ENEG(*some*)	*not some*


A
DUAL(I)
INEG(E)
INEG(A)
ENEG(I)
E
DUAL(A)
I
ENEG(E)
ENEG(A)
INEG(I)
DUAL(E)


all S are P
not some S are not P
no S are not P
all S are not P
not some S are P
no S are P
not all S are not P
some S are P
not no S are P
not all S are P
some S are not P
not no S are not P

Analogous story for the modal square





- interaction between duality and lexicalization
  - the square has 4 corners (Klein 4-group)
  - each corner has only 3 primitive formulations (lexicalization constraint)

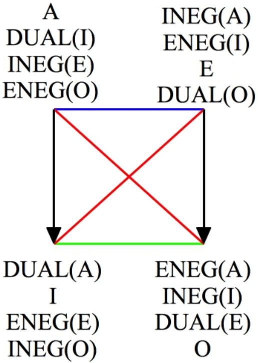
- the A-, I- and E-corner
  - primitive lexicalization
  - duality-theoretic variants of the **two** other primitively lexicalized corners

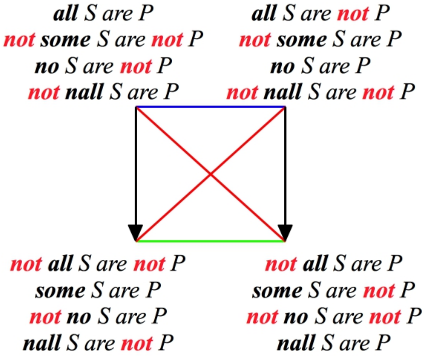
- the O-corner
  - no primitive lexicalization
  - duality-theoretic variants of the **three** other corners

- lexicalization has effects on **all** corners of the square (not just O)

A thought experiment

- what if O did have a primitive lexicalization, e.g. ***nall***?
- each of the four corners would have four equivalent formulations:
  - one primitive lexicalization
  - duality-theoretic variants of the three other corners





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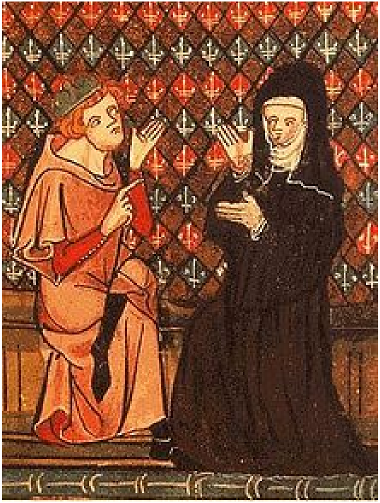
- individual authors:
  - **Peter Abelard** 1079 – 1142
  - William of Sherwood 1205 – 1270
  - **Peter of Spain** 1205 – 1277
  - Thomas Aquinas 1225 – 1274
  - William of Ockham 1287 – 1347
  - **John Buridan** 1300 – 1360
  - John Wyclif 1330 – 1384
  - Antoine Arnauld & Pierre Nicole (Port-Royal) 1662
  - Jacques Maritain (neo-Thomism) 1882 – 1973
- special topic of interest: **mnemonics**
  - mnemonic words for the square's four corners
  - mnemonic verses for the equipollences
  - mnemonic verses for the Aristotelian/duality interplay

Peter Abelard (ca. 1079 – 1142)



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- *Dialectica* (ed. L. M. de Rijk, 1956)
- discussion of modal logic (singular propositions)
- four *ordines propositionum*:

'possibile est Socratem esse album'	*'non impossibile est Socratem esse album'*	*'non necesse est Socratem non esse album*
'non possibile est Socratem esse album'	*'impossibile est esse album'*	*'necesse est non esse album'*
'possibile est Socratem non esse album'	*'non, impossibile est Socratem non esse album'*	*'non necesse est Socratem esse album'*
'non possibile est Socratem non esse album'	*'impossibile est Socratem non esse album'*	*'necesse est Socratem esse album'.*

- **equivalence**: *Sunt enim omnes cuiuslibet ordinis propositiones ad se aequipollentes*
- **contradiction**: *Et sunt quidem propositiones secundi dividentes cum propositionibus primi, et quarti cum tertii*
- **subalternation**: *Inferunt autem propositiones quarti propositiones primi, sed non convertitur; et propositiones secundi propositiones tertii, sed non convertitur*

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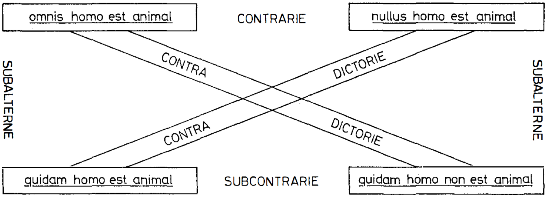
- Abelard had all the ingredients for the **purely modal** square (i.e. singular propositions, no quantifiers):
  - the four sets of three **equivalent propositions**
  - the Aristotelian **relations** between (the propositions in) those sets
  - the square as an actual two-dimensional **diagram**
    - square for the quantifiers in *Glossae super Peri Hermeneias*
    - square for the 'binary' quantifiers (*both*, *neither*, etc.) in the *Dialectica*

- however, as far as we know, he never drew the modal square with three equivalent propositions per corner

- Abelard tried to extend his system to **quantified modal propositions**, but those attempts are "rather confused" (Lagerlund 2000)

- Abelard's **quantifier square** cannot have three propositions per corner:
  - e.g. *some not* and *not all* are not logically equivalent for Abelard
  - the former has existential import, the latter does not

- *Summulae Logicales* (ed. L. M. de Rijk, 1973):
  quantifier square with one proposition per corner



- *si alicui signo **preponatur negatio**, equipollet suo **contradictorio***
- Peter's (only) example:
  - ***non omnis** homo currit*
  - ***quidam** homo **non** currit*

- *si alicui signo universali **postponatur negatio**, equipollet suo **contrario***
- one of Peter's examples:
  - ***omnis** homo **non** est animal*
  - ***nullus** homo est animal*

- *si alicui signo universali vel particulari **preponatur et postponatur negatio**, equipollet suo **subalterno***
- one of Peter's examples:
  - ***non omnis** homo **non** currit*
  - ***quidam** homo currit*

- combine:
  - the quantifier square (with one proposition per corner)
  - the rules for duality in the quantifier square
- Peter had all the resources to draw a quantifier square with three equivalent propositions per corner
- however, as far as we know, he never actually did so
- in some manuscripts of the *Summulae*, we find **mnemonic versions** of the rules as well as their results:
  - *Prae contradic, post contra, prae postque subalter*
  - *non omnis – quidam non; omnis non quasi nullus;*
    non nullus – quidam; sed nullus non valet omnis;
    non aliquis – nullus; non quidam non valet omnis;
    (non alter – neuter; neuter non prestat uterque.)

- verse for the rule (*Prae contradic, post contra, prae postque subalter*)
  - also in William of Sherwood, *Introductiones in Logicam*
  - also in John Wyclif, *Tractatus de Logica*
- verse for the results: different (clearer!) version in Sherwood
  - *Equivalent omnis, nullus non, non aliquis non.*
    Nullus, non aliquis, omnis non equiparantur.
    Quidam, non nullus, non omnis non sociantur.
    Quidam non, non nullus non, non omnis adherent.
- 12th and 13th century: “a veritable craze for versifying” (Paetow 1910)
- the *Summulae*'s “greater success may be due to the fact that it contains more and better mnemonic verses than William of Shyreswood's work.” (Kneale and Kneale 1964)

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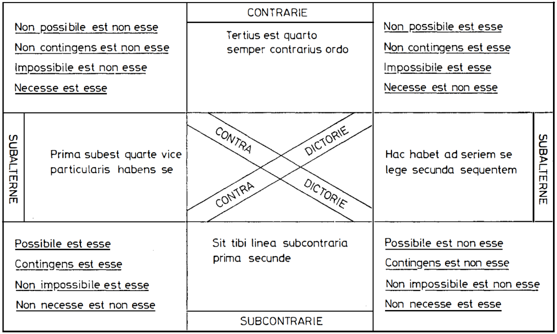
- recall the following rule from Peter:
  *si alicui signo **universali** postponatur negatio, equipollet suo **contrario***
- one might claim that Peter has forgotten the analogous rule:
  *si alicui signo **particulari** postponatur negatio, equipollet suo **subcontrario***
- given the non-lexicalization of the O-corner, the latter rule is trivial
- first rule: useful information about Latin/English
  - INEG(A) = *omnis non* = *nullus* = E
  - INEG(A) = *all not* = *no* = E
- second rule: trivial
  - INEG(I) = *quidam non* = *quidam non* = O
  - INEG(I) = *some not* = *some not* = O

Peter of Spain on the modal square

- Peter draws a modal square with **four** equivalent propositions per corner
- no need to differentiate between the first two in each corner:
  - in terms of *possibile* and *contigens*
  - *'contingens' convertitur cum 'possibili'*
- essentially: modal square with **three** equivalent propositions per corner



(only singular modal propositions;
no quantified modal propositions)

- *purpurea*, *amabimus*, *illiace*, *edentuli*
  - each word stands for a **corner** of the modal square (with its four equivalent propositions)
  - each syllable stands for a **modality** (cf. next slide)
  - each vowel stands for a **combination of negations** (cf. next slide)
- contrast with the more well-known *barbara*, *celarent*, etc.:
  - each word stands for a **syllogism** (three non-equivalent propositions)
  - each syllable stands for a **proposition** (premise/premise/conclusion)
  - each vowel stands for a **quantifier** (AEIO convention)
- popular throughout history:
  - Peter of Spain
  - William of Sherwood
  - (Pseudo-)Aquinas
  - Port-Royal Logic
  - Jacques Maritain and other neo-Thomists

Mnemonic terms for the corners of the modal square

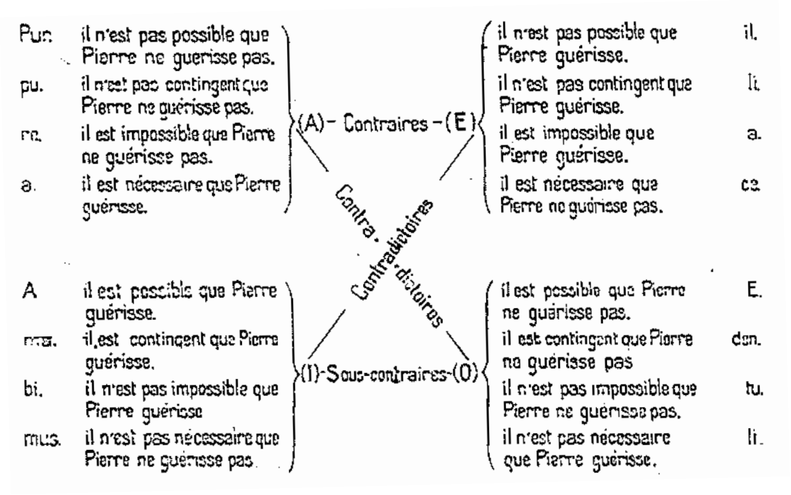
- *purpurea*, *amabimus*, *illiace*, *edentuli*

- the **order** of the syllables is significant:
  - syllable 1 $\sim$ a proposition containing *possibile*
  - syllable 2 $\sim$ a proposition containing *contingens*
  - syllable 3 $\sim$ a proposition containing *impossibile*
  - syllable 4 $\sim$ a proposition containing *necesse*

  vowels 1 and 2 always coincide!

- independent **convention** for the vowels:

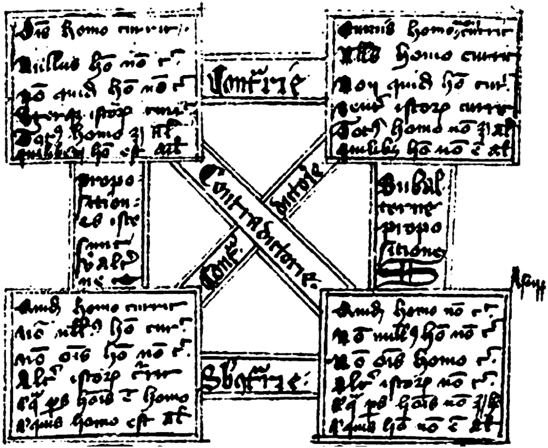
	Klein 4-group:
A: no negations at all	O
E: negation after the modality	INEG(O)
I: negation before the modality	ENEG(O)
U: negation before and after the modality	DUAL(O)


Pur. il n'est pas possible que Pierre ne guerisse pas.
pu. il n'est pas contingent que Pierre ne guérisse pas.
rn. il est impossible que Pierre ne guérisse pas.
a. il est nécessaire que Pierre guérisse.
il n'est pas possible que Pierre guérisse. il.
il n'est pas contingent que Pierre guérisse. li.
il est impossible que Pierre guérisse. a.
il est nécessaire que Pierre ne guérisse pas. ce.
(A) – Contraires – (E)
Contra-dictoires
Contradictoires
A il est possible que Pierre guérisse.
ma. il est contingent que Pierre guérisse.
bi. il n'est pas impossible que Pierre guérisse
mus. il n'est pas nécessaire que Pierre ne guérisse pas.
il est possible que Pierre ne guérisse pas. E.
il est contingent que Pierre ne guérisse pas den.
il n'est pas impossible que Pierre ne guérisse pas. tu.
il n'est pas nécessaire que Pierre guérisse. li.
(I)-Sous-contraires-(O)

KU LEUVEN

John Buridan on the quantifier square

- *Summulae de Dialectica* (trans. G. Klima, 2001):
  quantifier square with six propositions per corner



- Buridan has a quantifier square with **six** propositions per corner

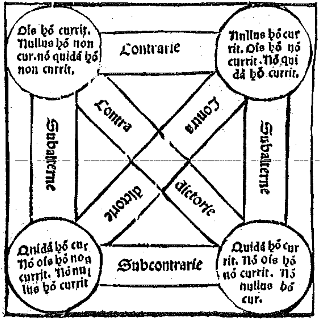
- consider, for example, the A-corner:
  - ***Omnis*** *homo currit*
  - ***Nullus*** *homo* ***non*** *currit*
  - ***Non quidam*** *homo* ***non*** *currit*

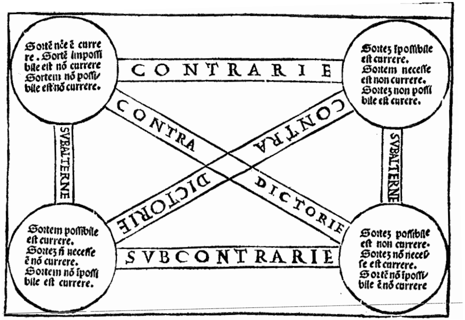
  - *Uterque istorum currit*
  - *Totus homo est animal*
  - *Quilibet homo est animal*

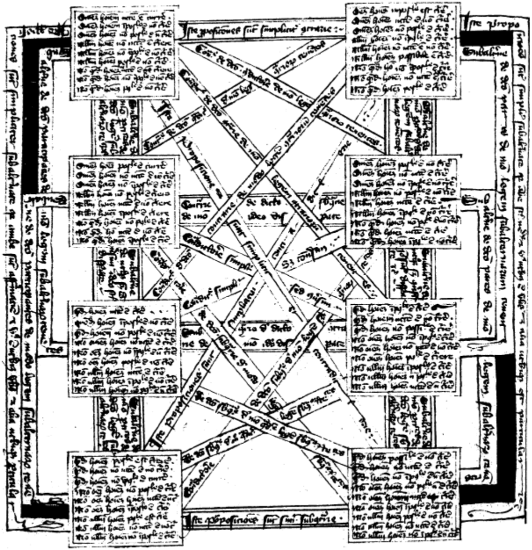
- the last three are only relevant from a broader linguistic perspective: demonstratives, 'binary' quantifiers, mass nouns, free choice

- quantifier square with **three** equivalent propositions per corner!

- *Compendium totius Logicae* = later summary of the *Summulae* (by John Dorp in 1499, so 150 years after Buridan's death)
- the *Compendium* contains
  - a quantifier square with three equivalent propositions per corner
  - a modal square with three equivalent propositions per corner







- first CLAW/DWMC symposium (Demey & Steinkrüger 2017):
  - Buridan's octagon can be understood as capturing the interaction between a quantifier square and a modal square
  - Buridan himself was already well aware of this

octagon	=	quantifier square	×	modal square
9 propositions per corner		3 propositions per corner	×	3 propositions per corner

- first symposium: focus on $9 = 3 \times 3$
- today: why 3 to begin with?

	quantifier square with 3 equivalent propositions per corner	modal square with 3 equivalent propositions per corner
Peter Abelard	no!	no, but can be constructed
Peter of Spain	no, but can be constructed	yes
John Buridan	yes	yes

Thank you!

More info: www.logicalgeometry.org